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EVALUATION OF STUDENTS CONCEPTUAL UNDERSTANDING IN THE COURSE MULTIPLE VARIABLE CALCULUS

EVALUASI PEMAHAMAN KONSEPTUAL MAHASISWA DALAM MATA KULIAH KALKULUS PEUBAH GANDA

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Abstract

Calculus of Multiple Variables is a core course in the Mathematics Education study program and plays a crucial role in developing students advanced mathematical thinking skills. Success in this course relies heavily on mastery of the prerequisite material, particularly the concepts of derivatives and integrals of functions of one variable. Therefore, understanding mathematical concepts is a fundamental aspect that determines students’ ability to understand the interrelationships between topics and solve problems systematically and logically. This study aims to describe students' conceptual understanding of multivariable calculus. The approach used is a descriptive approach with a purposive sampling technique, namely selecting research subjects based on certain criteria in the form of the highest and lowest levels of mathematical concept understanding. The results of the study indicate that students conceptual understanding varies across different materials. In the partial derivative material, students were not yet fully able to accurately represent the solution concept, especially in applying the rules for derivatives of exponential functions, so the results obtained did not conform to mathematical principles. These errors are conceptual in nature and indicate a weak grasp of certain basic concepts. However, in the material on derivatives of products of two functions and triple integrals, students demonstrated goog conceptual understanding. They were able to select, use, and apply solution procedures appropriately and systematically. This finding indicates that despite weaknesses in certain concepts, students still possess quite good procedural and conceptual abilities in the context of other problems in polynomial calculus.

Keywords : Evaluation, Conceptual Understanding, Multiple Variable Calculus.

Abstrak

Kalkulus peubah banyak merupakan salah satu mata kuliah inti dalam program studi Pendidikan Matematika yang berperan penting dalam mengembangkan kemampuan berpikir matematis tingkat lanjut mahasiswa. Keberhasilan dalam mempelajari mata kuliah ini sangat bergantung pada penguasaan materi prasyarat, khususnya konsep turunan dan integral pada fungsi satu variabel. Oleh karena itu, pemahaman konsep matematis menjadi aspek fundamental yang menentukan kemampuan mahasiswa dalam memahami keterkaitan antar topik serta menyelesaikan permasalahan secara sistematis dan logis. Penelitian ini bertujuan untuk mendeskripsikan pemahaman konsep mahasiswa pada mata kuliah

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kalkulus peubah banyak. Pendekatan yang digunakan adalah pendekatan deskriptif dengan teknik purposive sampling, yaitu memilih subjek penelitian berdasarkan kriteria tertentu berupa tingkat pemahaman konsep matematis tertinggi dan terendah. Hasil penelitian menunjukkan bahwa pemahaman konsep mahasiswa bervariasi pada materi yang berbeda. Pada materi turunan parsial, mahasiswa belum sepenuhnya mampu merepresentasikan kembali konsep penyelesaian secara tepat khususnya dalam menerapkan aturan turunan fungsi eksponen, sehingga hasil yang diperoleh belum sesuai dengan kaidah matematis. Kesalahan ini bersifat konseptual dan menunjukkan lemahnya penguasaan konsep dasar tertentu. Namun demikian, pada materi turunan hasil kali dua fungsi dan integral lipat tiga mahasiswa menunjukkan pemahaman konseptual yang baik. Mahasiswa mampu memilih, menggunakan, dan menerapkan prosedur penyelesaian secara tepat dan sistematis. Temuan ini mengindikasikan bahwa meskipun terdapat kelemahan pada konsep tertentu, mahasiswa tetap memiliki kemampuan prosedural dan konseptual yang cukup baik pada konteks permasalahan lain dalam kalkulus peubah banyak.

Kata Kunci : Evaluasi, Pemahaman Konsep, Kalkulus Peubah Banyak.

1. INTRODUCTION

The faculty of Teacher Training and Education (FKIP) at Batanghari University is a higher education institution with a strategic role in producing future professional educators who will be responsible for educating and shaping the character of their students. Through a structured educational and learning process, FKIP not only equips students with mastery of scientific knowledge and pedagogical skills but also inculcate the moral values, ethics, and social responsibility that are essential to the teaching profession.

Multiple variable calculus is a core course in the Mathematics Education study program that plays a crucial role in equipping students with advanced mathematical thinking skills. This course examines functions involving more than one variable, thus requiring more complex analytical skills that basic calculus. Topics covered in Multiple variable calculus include partial derivatives, total differentials, gradients, multiple integrals (double and triple integrals), extreme points, and various applications in geometry, physics, economics, and other applied sciences. These materials are not only procedural in nature, but also emphasize conceptual understanding and the interrelationships between concepts.

As an advanced calculus, Multiple variable calculus relies heavily on mastery of prerequisite material, particularly the concepts of derivatives and integrals for functions of one variable. Salsabila & Heni (2022) also stated that understanding the course requires having studied and mastered the concepts of derivatives and integrals. If students are familiar with the concepts of derivatives and integrals, they will find it easier to learn, understand, and solve problems in course effectively. This understanding serves as the foundation for studying partial derivatives and multiple integrals, which essentially extend the concepts of derivatives and integrals into higher dimensions. Without mastery of these basic concepts, students tend to have difficulty grasping the geometric and analytical meaning of the material and rely solely on memorizing formulas without in depth understanding.

Thus, understanding mathematical concepts is a crucial aspect of studying Multiple variable calculus. Students who have a good graps of derivatives and integrals will more easily

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follow the flow of the material, understand the relationships between topics, and solve problems systematically and logically. This not only impacts students' academic success in the course but also contributes to their readiness as future mathematics educators, able to explain concepts meaningfully to students at the next level.

According to Tsurayya & Nur (2021), mastery of mathematical concepts and principles is a prerequisite for successful mathematics learning, leading to higher levels of learning. This is because simple concepts are connected to more complex ones. This is in line with Fitria (2020) opinion that when students misuse concepts to solve a problem, errors in procedures, algorithms, and solutions will occur. Therefore, conceptual understanding is a skill that students must master in learning mathematics.

Based on initial observations by looking at the results of the exercise which is a question of understanding mathematical concepts with indicators restate the concept, it looks like the following image.

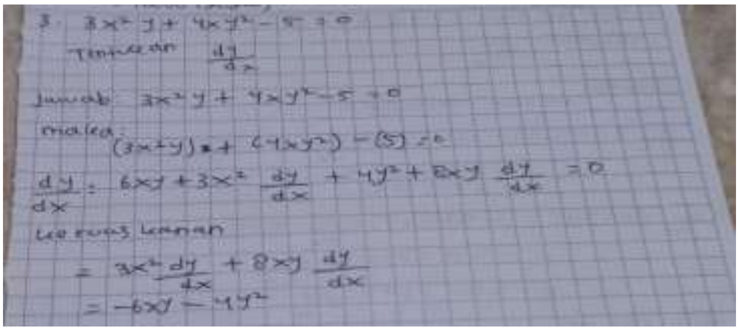

Figure 1. Example of Student Answers on the Indicators Restate the Concept

Based on Figure 1 above, the results of students conceptual understanding associated with the restating concept indicator can be said to be still low. The problem is an implicit derivative problem with direct differentiation. In the initial stages of solving, students should first decompose the equation from an implicit form ti a more explicit form or clearly perform the differentiation process by differentiating each term with respect to the variables x and y. However, the solution steps shown do not yet reflect students conceptual understanding of the implicit differentiation process. This is evident in the lack of explanation of partial or implicit derivatives for terms in the initial equation. Furthermore, in the third step students directly write $\frac{dy}{dx}$ without explicitly indicating which terms or expressions in the previous steps were derived to produce this form. As a result, the flow of mathematical understanding becomes opaque and makes it difficult to assess the consistency of the application of the learned derivative concept.

Several previous studies have examined students' conceptual understanding in calculus courses. One such study is by Simorangkir & Ferry (2022), which analyses students conceptual understanding errors in the material on double integrals over non-rectangular areas. This study focused on identifying the types of conceptual errors students made in determining the boundaries of the region of integration and in applying the procedural calculations for double integrals. Furthermore, research conducted by Salsabila & Heni (2022) entitled analysis of

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students understanding of mathematical concepts in online learning in the multiple variable calculus course. This research focuses on assessing the level of students understanding of mathematical concepts in the context of online learning, specifically in the multiple variable calculus course. The main focus of the research includes students' ability to understand definitions, apply concepts, and relate various mathematical representations related to multiple variable calculus material. The results of the study indicate that the implementation of online learning presents its own challenges to students conceptual understanding, especially in understanding abstract concepts that require visualization and high-level mathematical reasoning.

Based on a review of previous studies, it can be concluded that most research on students mathematical conceptual understanding in multiple variable calculus courses still focused on analyses certain conceptual errors or on certain learning contexts, such as online learning. These studies generally emphasize identifying weaknesses in students understanding of specific materials or learning situations, but have not comprehensively evaluated students mathematical understanding based on systematic indicators of mathematical understanding.

Therefore, the innovation in this research lies in a more comprehensive and structured evaluation of students mathematical understanding. This research not only identifies errors or the level of conceptual mastery, but also evaluates students mathematical understanding based on indicators of understanding such as the ability to restate concepts, apply concepts in problem solving, and relate various mathematical representations. Thus, this research is expected to provide a more comprehensive picture of the state of students mathematical understanding and serve as a basis for designing more effective learning strategies in Multiple Variable Calculus courses.

2. RESEARCH METHOD

This research is categorized as qualitative research. According to Sugiono (2017) qualitative research is a research method used to examine the natural conditions of objects, where the researcher acts as the primary instrument in data collection. The approach used in this study is a descriptive approach. The descriptive approach aims to systematically, factually, and accurately describe the facts, characteristics and relationships between the phenomena being studied. In the context of this research, the descriptive approach is used to describe the level of understanding of mathematical concepts among students in the Multiple Variable Calculus course, as well as the actual conditions at the time of the research.

The data sources for this study were students in the Multiple Variable Calculus course. From all these data sources, the researchers selected research subjects based on a previously administered mathematical concept understanding test. The indicators of mathematical concept understanding used in this study can be seen in table 1 below.

Table 1. Concepts Understanding Indicators

No	Concepts Understanding Indicators	Question Items
1	The ability to restate a concept	1b
2	Ability to present concepts in various forms of mathematical	2

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	representation	
3	The ability to develop necessary or sufficient conditions for a concept	1a
4	The ability to use, utilize and select certain procedures	3

(Modification of the Ministry of National Education, 2006)

Subject selection was conducted to obtain respondents who were in line with the research objectives. To determine truly relevant research subjects, a purposive sampling technique was used. According to Nikolopoilou (2023), purposive sampling is a non-probability method in which sample units are selected because they possess the characteristics required for the study. In the context of this research, the application of purposive sampling means that the researcher sets certain criteria, namely the highest and lowest scores or levels of mathematical concept understanding as conditions for inclusion in the study, then selects only students who meet these criteria as research subjects. Thus, the selected subjects are expected to provide representative information on the phenomena to be studied in the Multiple Variable Calculus course.

3. RESULT AND DISCUSSION

The test results obtained in this study were used to analyses the level of mathematical concept understanding based on the answers given by each student. The mathematical concept understanding test used in this study consisted of four questions, each designed to represent a single indicator of mathematical concept understanding. Thus, the test result allowed researchers to systematically and measurably assess student achievement on each indicator of conceptual understanding.

a. Student conceptual understanding of Indicators the ability to restate a concept

The first question in the conceptual understanding test, spesifically part b is a question related to partial derivatives, with the indicator being the ability to restate the concept. The student with the highest score (JMST) was able to solve the problem by determining the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ but the overall answer was not correct. The process and result of the solution shown by JMST can be observed in Figure 2 below.

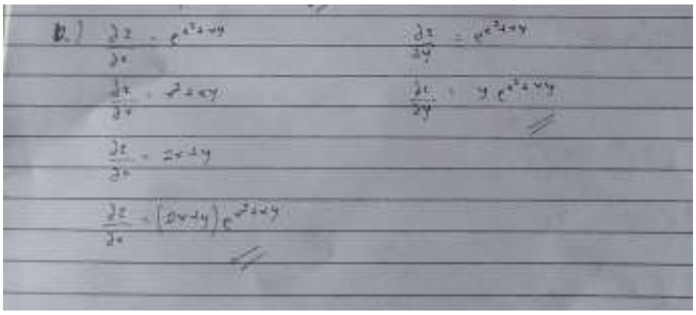

Figure 2. JMST Indicator Restates the Concept

Based on Figure 2 above, the final solution for the partial derivative $\frac{\partial z}{\partial x}$ can still be simplified to $\frac{\partial z}{\partial x} = z(2x + y)$. However, the solution to the partial derivative $\frac{\partial z}{\partial y}$ obtained is still

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not correct because the application of the derivative concept to the variable y is not done correctly. This indicates a misunderstanding or restatement of the concept of partial derivatives for the variable. Different results were shown by the answers of the students with the lowest scores (JMSR). The student wrote down the partial derivative solution for the variable x and the variable y. However, the answers provided were not able to demonstrate the ability to restate or accurately represent the concept of solving partial derivatives. This indicates that the students did not yet understand the basic concept of partial derivatives despite having attempted to write down the solution form.

b. Students Conceptual Understanding of Indicators Ability to Present Concepts in Various Forms of Mathematical Representation

This section presents the results of research on students understanding og mathematical concepts, based on the indicator of their ability to present concepts in various mathematical representations. The evaluation focuses on how students express and present mathematical concepts through various representations and other mathematical forms. This indicator is measured in question number two, which deals with the material on implicit function derivatives with direct differentiation. The process and results of the solution shown by the JMST can be observed in Figure 3 below.

Figure 3. JMST Indicators Present Concepts in Various Forms of Mathematical Representation

Based on the JMST answer, it is explained that the student has demonstrated procedural understanding in solving implicit derivatives using the direct differential method. This can be seen from their ability to represent each derivative with respect to the x and y variables, which shows that students have understood the basic principles of implicit differentiation. However, there is a conceptual error in the application of the rules for the derivative of exponential functions. This incorrect representation of the derivative of exponential functions causes the differentiation process to be inconsistent with mathematical rules, resulting in an incorrect final result. Furthermore, based on the JMSR answers, it can be said that students also applied direct differentiation to the given equations. This approach demonstrates a general understanding of implicit differentiation techniques. However, the problem that emerged was similar to the JMST answer, namely the inaccuracy in representing the derivative of the exponential function. This conceptual error indicates that students have not yet mastered the rules of derivatives of exponential functions in depth, so that both the process and the results of the differentiation obtained do not meet the expected mathematical accuracy.

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c. *Students Conceptual Understanding of Indicators Ability to Develop Necessary or Sufficient Conditions for A Concept*

The first question in the conceptual understanding test specifically part A is a question related to partial derivative material, with the indicator being the ability to develop necessary or sufficient conditions for a concept. The process and results of the solution shown by JMSR can be observed in Figure 4.

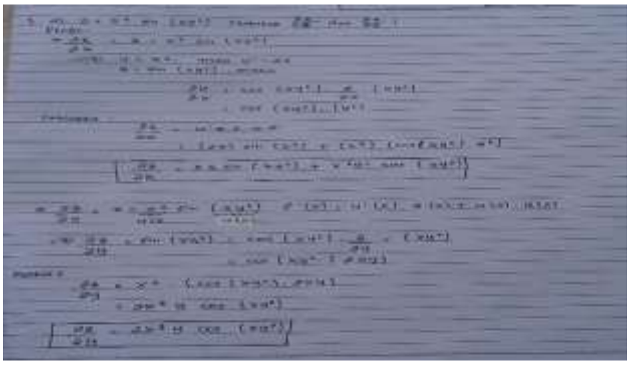

Figure 4. JMSR Indicator Develops Necessary or Sufficient Conditions of A Concept

Based on JSMR answers, it can be seen that students solve partial derivatives by first outlining the necessary conditions, namely applying the derivative formula for the product of two functions. This approach shows that students have understood the basic concept of partial differentiation, especially in dealing with functions expressed as the product of two functions that depend on certain variables. Furthermore, based on the JMST answers, it can be seen that the students also used the same approach in solving the problem. This indicates a consistency in conceptual understanding between the two students in applying the product rule to the context of partial derivatives. Students have demonstrated excellent abilities in developing necessary and sufficient conditions for the concept of the derivative of the product of two functions as a basis for continuing the process of solving partial derivatives. Furthermore, in selecting the function examples $u(x)$ and $v(x)$, students have done it precisely and systematically. The process of deriving each function is also carried out in accordance with the applicable differentiation rules. Accuracy in determining the examples and carrying out the derivation procedure has a positive impact on the accuracy of the final results obtained, so that the partial derivatives $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ can be determined correctly and in accordance with applicable mathematical principles.

d. *Students Conceptual Understanding of Indicators of Ability to Use, Utilize and Select Certain Procedures*

This section examines Students conceptual understanding of the ability to use, utilize, and select specific procedures in solving mathematical problems. The discussion focuses on how students determine relevant procedures, apply appropriate rules or formulas, and utilize learned concepts appropriately and systematically. By evaluating the process and results of student solutions, this section aims to identify the level of accuracy of procedure selection and

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the depth of conceptual understanding underlying the solution steps taken. The process and results of the solutions demonstrated by the JMSR can be observed in Figure 5.

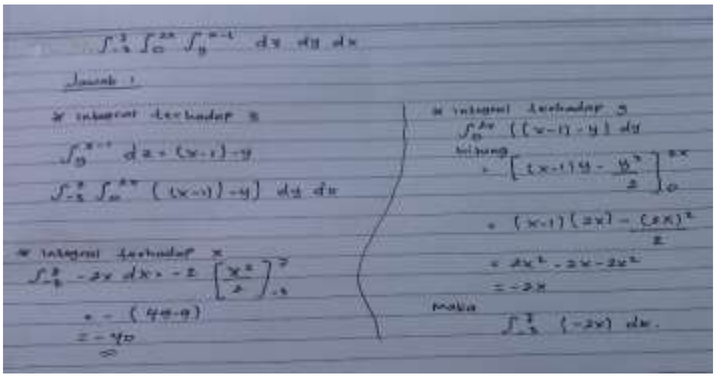

Figure 5. JMSR Indicators Using, Utilizing and Selecting Certain Procedures

Based on JMSR answer to question no. 3 which is related to triple integral material, it can be seen that students have demonstrated good abilities in using, utilizing, and selecting the right procedures in solving the problem. This is reflected in the students' accuracy in determining the sequence of the integration process, namely by starting the integration with respect to variable z then continuing with integration with respect to other variables until ending with the integration with respect to variable x. The choice of this integration sequence requires a good conceptual understanding, as it differs from the usual procedure that begins with the variable x. The students' ability to choose unconventional integral sequences shows that the students do not only rely on mechanical procedures but are also able to adapt the solution steps to the form of the integration region and the given function. Thus, students can apply the triple integral concept flexibly and according to the needs of the problem.

A similar approach was demonstrated by JMST students, who applied the solution procedure with a precise and consistent integration sequence. The similarity in approach between the two students indicates that both students have a good understanding of the triple integral concept, particularly in selecting and applying the appropriate integration procedure. Therefore, it can be concluded that both JMSR and JMST are able to correctly solve triple integral problems through selecting the right procedure and systematically applying the concept.

4. CONCLUSION

Based on this description, it can be concluded that students conceptual understanding showed varying results across different materials. For the partial derivative material, the answers given did not fully reflect students' ability to restate or accurately represent the solution concept. Inaccuracy in representing the derivative of an exponential function indicates a weakness in understanding the basic concept, so that the differentiation process and results obtained do not conform to applicable mathematical rules. This error is conceptual and indicates that students have not yet thoroughly mastered the rules for the derivative of an exponential function, despite having attempted to write down the solution steps.

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However, in other aspects the students conceptual understanding is strong. Both JMSR and JMST students demonstrated consistent understanding in applying the rule for the product of two functions, both in developing necessary and sufficient conditions as a basis for solving partial derivatives. Furthermore, in the triple integrals' topic both students were able to systematically select and apply appropriate solution procedures to produce correct answers. This demonstrates that despite weaknesses in understanding certain concepts, the students have demonstrated good skills in using and utilizing mathematical procedures in appropriate problem contexts.

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